

# Modeling the emergence of universality in color naming patterns

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**The empirical evidence that human color categorization exhibits some universal patterns beyond superficial discrepancies across different cultures is a major breakthrough in cognitive science. As observed in the World Color Survey (WCS), indeed, any two groups of individuals develop quite different categorization patterns, but some universal properties can be identified by a statistical analysis over a large number of populations. Here, we reproduce the WCS in a numerical model in which different populations develop independently their own categorization systems by playing elementary language games. We find that a simple perceptual constraint shared by all humans, namely the human Just Noticeable Difference (JND), is sufficient to trigger the emergence of universal patterns that unconstrained cultural interaction fails to produce. We test the results of our experiment against real data by performing the same statistical analysis proposed to quantify the universal tendencies shown in the WCS [Kay P & Regier T. (2003) *Proc. Natl. Acad. Sci. USA* 100: 9085-9089], and obtain an excellent quantitative agreement. This work confirms that synthetic modeling has nowadays reached the maturity to contribute significantly to the ongoing debate in cognitive science.**

computational cognitive science | natural categorization | multiagent modeling | complex systems | statistical physics

The finding that color naming patterns present some conserved features across cultures (1) is a milestone in the debate on the existence of universals in human categorization (2). The data collected in the World Color Survey (WCS) (3), extending the pioneering work by Berlin and Kay (1), provide empirical evidence in favor of the fact that categorization is not simply a matter of conventions, but rather depends on the physiological and cognitive features of the categorizing subjects, in contrast with previous theories according to which categories are arbitrarily defined by different cultures (4). Over the years, the existence of universals in color categorization has become generally accepted (2, 5–7), though it has been the subject of strong controversy, and a debate is still ongoing (8–12). Recently, however, a set of statistical tests have proved quantitatively that the WCS data do in fact contain clear signatures of universal tendencies in color naming, both across industrialized and nonindustrialized languages (13). In any case, the WCS maintains a central role and its data, as a fundamental (and almost unique) experimental repository, are still under constant scrutiny, as shown by the continuous flow of publications related to them (see, for instance, (13–18)).

Color categorization represents a case study in a wide debate on the origins, meanings, and properties of categorization systems (6, 7). In recent years, mathematical and computational models have been designed to explore the roles of various hypotheses concerning these issues, checking their implications in simplified, yet hopefully transparent, synthetic experiments (19). Generally speaking, a model can show whether its assumptions are internally coherent, whether they can produce the claimed effects, and above all, whether they are sufficient in principle to generate a given phenomenon. Although a model cannot

prove if those assumptions are empirically valid, it can test the available hypotheses, suggest new perspectives, and help to ask more focused questions (19). Color categorization has been used as a reference problem also in computational studies that investigate how much language and perceptually grounded categories influence each other and how a population of individuals establish a shared repertoire of categories. Pioneering work in this direction has shown that purely cultural negotiation in the form of iterated Language Games (20) can lead to a coevolution of categories and their linguistic labels (21, 22) in a population of individuals. This line of research has been subsequently extended, and complex systems methods have demonstrated that cultural interaction is able to yield a finite number of categories even when the perceptual space is continuum, as in the case of color perception (23). A different point of view has been adopted in the framework of the Iterated Learning Model (24, 25), in which a population is modeled as a chain of individuals, each learning from the output of the previous generation and providing the input to the subsequent one (26). In this context, it has been suggested that universals in categorization may originate from the presence of unevenly distributed salient color foci in the perceptual space (27). Finally, the Evolutionary Game Theory (28) approach has focused mainly on the roles played by different individual features (being linguistic, psychological, or physiological) on the shared color categorization (29), such as the influence of few abnormal observers on the whole categorization system (30).

Employing an *in silico* experiment, in this paper we show that cultural transmission can induce universal patterns in color categorization, provided that some basic properties of human perceptual system are considered. We generate “synthetic” languages via an agent-based model (23) that simulates a number of independent groups of interacting individuals. We identify universal patterns in color naming among groups whose members are endowed with the human Just Noticeable Difference (JND) function describing how the resolution power of human vision varies according to the frequency of the incident light. These results are tested against the experiment in which individuals perceive the spectrum homogeneously. Strikingly, following the same analysis of (13), we find that the difference between these two types of languages is in excellent agreement with the difference between the experimental and randomized data measured by Kay and Regier in their work based on the WCS (13). Such an agreement is remarkable, considering the rather minimal input introduced: except for the JND curve, our experiment is blind to any other properties of the real world or real human beings.

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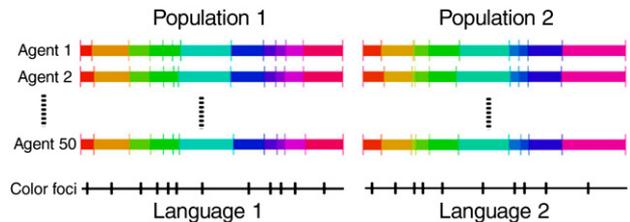
This article contains supporting information online at [www.pnas.org/cgi/content/full/0908533107/DCSupplemental](http://www.pnas.org/cgi/content/full/0908533107/DCSupplemental).

## The Category Game Model

The computational model used in this experiment, introduced in (23), involves a population of  $N$  artificial agents. Starting from scratch and without predefined color categories, the model dynamically generates, via a number of “games”, a pattern of linguistic categories for the visible light spectrum highly shared in the whole population. The model has the advantage of incorporating an extremely low number of parameters, basically the number of agents  $N$  and the JND curve  $d_{\min}(x)$  (detailed in the *Methods*), compared with its rich and realistic output.

For the sake of simplicity and not losing the generality for the purpose of analysis, color perception is reduced to a single analogical continuous perceptual channel, each stimulus being a real number in the interval  $[0, 1)$  that represents its normalized, rescaled wavelength. A categorization pattern is identified as a partition of the interval  $[0, 1)$  into subintervals, or perceptual categories. Individuals have dynamic inventories of form-meaning associations that link perceptual categories with their linguistic counterparts, i.e., basic color terms, and these inventories evolve through elementary language games (20). At each time step, two players (a speaker and a hearer) are chosen randomly from the population and a scene of  $M \geq 2$  stimuli is presented to them. Any two of these stimuli cannot appear at a distance smaller than  $d_{\min}(x)$ , where  $x$  is the value of either of the two. In this way, the JND is implemented in the model. Based on the presented stimuli, the speaker discriminates the scene, if necessary refines its perceptual categorization, and utters the color term associated to one of the stimuli (the topic). The hearer tries to guess the topic, and based on the outcome (success or failure) both individuals rearrange their form-meaning inventories (further details of this process are given in the *Methods*, and the complete algorithm of the Category Game is provided in *SI Text*). New color terms are invented every time a new category is created for the purpose of discrimination, and these terms can diffuse through the population in successive games. At the beginning, all individuals have only the perceptual category  $[0, 1)$  with no associated name.

During the first phase of the evolution, the pressure of discrimination makes the number of perceptual categories increase: at the same time, many different words are used by different agents for some similar categories. This kind of synonymy reaches a peak and then dries out, in a way similar to the well-known Naming Game (31–33). When on average only one word is recognized by the whole population for each perceptual category, the second phase of the evolution intervenes. During this phase, words expand their reference across adjacent perceptual categories, joining these categories to form unique “linguistic categories”. The coarsening of these categories becomes slower and slower, with a dynamic arrest analogous to the physical process in which supercooled liquids approach the glass transition (34). In this long-lived, almost stable phase, usually after  $10^4$  games per player, the linguistic categorization pattern has a 90% to 100% degree of sharing among individuals and remains stable for a long plateau phase whose duration diverges with the population size (lasting for instance  $10^5 \sim 10^6$  games per player (23) for the population sizes considered). We consider the pattern corresponding to this plateau phase as the “final pattern” generated by the model, i.e. the most relevant for comparison with human color categories. If one waits for a much longer time, the number of linguistic categories will drop. This unrealistic effect is caused by the extremely slow diffusion of category boundaries,\* which is due to finite-size effects, occurring at longer and longer times as the population size increases. Nonetheless, since the comparison with the real world is much less accessible on such a long time-scale, we are not interested in the behavior of the model in this



**Fig. 1.** An example of the results from the simulations of two different populations with the human JND ( $d_{\min}(x)$ ) function. After  $10^4$  games, the pattern of categories and associated color terms are stable throughout the population. Different individuals in one population have slightly different category boundaries, but the agreement is almost perfect (larger than 90%). As for each category, a focal color point is defined as the average of the midpoints of the same category across individuals in the population. Different populations may develop different final patterns.

phase (see also the discussion on the relevance of preasymptotic states in categorization models in (19)).

The shared pattern in the long-lived, stable phase between  $10^4$  and  $10^6$  games per player is the focus of the experiment described in the following section, see Fig. 1 for an example. It is remarkable, as already observed in (23), that the number of linguistic color categories achieved in this phase is of the order of  $20 \pm 10$ , even if the number of possible perceptual categories ranges between 100 and  $10^4$  and the number of agents ranges between 10 and 1,000. For this reason, the mechanism of spontaneous emergence of linguistic categories in this model is relevant for the problem of linguistic categorization in continuous spaces (such as the perceptual space of colors) where no objective boundaries are present.

## The Numerical World Color Survey

The aim of our study is to replicate *in silico* the WCS by performing a Numerical World Color Survey (NWCS). To this purpose, we generate “worlds” made of isolated populations. Each population is the outcome of a run of the model with  $N$  individuals, and each “world” collects 50 such populations (the logical scheme of this experiment is shown in Fig. 2). The sequence of games in each run is random, thus making each evolution history unique and each final shared pattern of linguistic color categories different across populations. Two classes of “worlds” are created: “human worlds” are obtained by endowing the individuals with the human JND function, while “neutral worlds” are obtained by using a uniform JND, i.e.,  $d_{\min}(x) = 0.0143$ , which is the average value of the human JND (as it is projected on the  $[0, 1)$  interval).

In all cases, as showed in (23), each population develops a shared repertoire of roughly 10–20 linguistic categories in the stable phase: this number of linguistic categories is weakly dependent on  $N$ , and the value  $N = 50$  is a good compromise to obtain representative results without increasing too much the length of simulations. The robustness of our findings with respect to different population sizes, as well as to changes in other parameters of the model, is well verified, as reported in *SI Text*. The hypothesis we test here is that the similarity between the linguistic patterns developed in the “human worlds” is higher on average than the one observed in the “neutral worlds”. To this aim, we compute, for each “world”, the quantity  $D$  defined to measure the dispersion of patterns of color terms in the WCS (13) (see the *Methods* for its definition). Following the same procedure used in the WCS, we define the representative point of each linguistic category as its central point (see *SI Text* for the details of our procedure).

The analysis of the WCS data has showed that the patterns collected in the survey are less dispersed (i.e., more clustered) than their randomized counterparts, thus proving the existence of universality in color categorization. Our simulations consider

\*In the Category Game, categories can be equivalently described in terms of boundaries or prototypes, without any difference (23).



of the randomized datasets in Fig. 3A of (13). To be consistent with the above procedure, the abscissa is normalized by the value of the nonrandomized dataset and the frequencies are rescaled by the width of the bins.

Fig. 3 illustrates two remarkable results. First, the Category Game Model informed with the human  $d_{\min}(x)$  JND curve produces a class of “worlds” that has a dispersion lower than and well distinct from that of the class of “worlds” endowed with a nonhuman, uniform  $d_{\min}(x)$ . Second, the ratio observed in the NWCS between the average dispersion of the “neutral worlds” and the average dispersion of the “human worlds” is  $D_{\text{neutral}}/D_{\text{human}} \sim 1.14$ , very similar to the one observed between the randomized datasets and the original experimental dataset in the WCS.

In summary, the color categories emerged in the “human worlds” in the NWCS are significantly less dispersed than those emerged in the “neutral worlds”,<sup>†</sup> and this difference agrees quantitatively with that observed in the WCS, where human languages are considered. Also, in *SI Text*, we show that even changing the population size, the number  $M$  of objects in a scene, or the time (measured in terms of games per agent) at which the categorization system is observed, our results agree quantitatively with the ones reported in (13), within an error of at most 10%. Considering the huge degree of reduction and simplification that separates the Category Game Model from the human language, this finding is definitely remarkable.

## Discussion and Conclusion

Through an *in silico* experiment, we have shown that independent groups of interacting agents incorporating a single human, presumably biological, constraint (the human JND function) end up developing categorization systems that exhibit universal properties similar to those observed in the WCS. We have also pointed out that replacing the human JND function with the uniform JND produces a similar effect of an a posteriori randomization on the WCS data, which is illustrated by the quantitative agreement between the results obtained in our experiment and those extracted from the WCS data in (13). Taken as a whole, our findings corroborate previous evidence in favor of the existence of universality in color naming systems (13, 16). Moreover, they suggest that the human perceptual (i.e., visual) system can be responsible for, or at least involved in, the emergence of such universal properties of categorization systems (17, 36). More precisely, the NWCS proves that purely cultural interaction among individuals sharing an elementary perceptual bias is sufficient to trigger the emergence of the universal tendencies observed in human categorization. Although the bias does not affect the properties of the shared categorization system in a deterministic way, it is responsible for subtle similarities that can be revealed by a statistical analysis on a large number of different populations.

Our work also demonstrates that computational approaches have nowadays reached good maturity, since the multiagent model presented here (i) incorporates straightforwardly a real feature of human perceptual system (i.e. the human hue-JND), and produces results (ii) testable against and (iii) in quantitative agreement with the empirical data. In addition, since the model is simply designed, there is a particularly transparent connection between the incorporated hypothesis and the generated results. Future work can further enrich the present picture, for example by considering a multidimensional perceptual channel or the impact of differences among individuals on the emergent category system, in the spirit of (30). Moreover, the close tie between our study and human perception, as discussed above, can help to inspire unique experiments and to design and analyze human or artificial communication systems (37, 38). In conclusion, we

believe that the results presented here not only contribute to the debate on the origins of universals in categorization, but also stimulate new efforts towards the growth of a computational cognitive science.

## Methods

**The WCS and the Dispersion Measurement.** A first survey was run on 20 languages in 1969 by P. Kay and B. Berlin (1). From 1976 to 1980, the World Color Survey was conducted by the same researchers along with W. Merrifield. Since 2003, the data have been made public on the website <http://www.icsi.berkeley.edu/wcs>. These data concern the basic color categories in 110 languages without written forms and spoken in small-scale, nonindustrialized societies. On average, 24 native speakers of each language were interviewed. Each informant had to name each of 330 color chips produced by the Munsell Color Company that represent 40 gradations of hue and maximal saturation, plus 10 neutral color chips (black-gray-white) at 10 levels of value. These chips were presented in a predefined, fixed random order.

Recently, Kay and Regier (13) performed the following statistical analysis. After a suitable transformation, they identified the most representative chip for each color in each language as a point in the CIEL\*a\*b color space, where an Euclidean distance is defined. In order to investigate whether these points are more clustered across languages than would be expected by chance, they defined a dispersion measure on this set of languages  $S_0$

$$D_{S_0} = \sum_{l, l' \in S_0} \sum_{c \in l} \min_{c' \in l'} \text{distance}(c, c'),$$

where  $l$  and  $l'$  are two different languages,  $c$  and  $c'$  are two basic color terms respectively from these two languages, and  $\text{distance}(c, c')$  is the distance between the points in CIEL\*a\*b space that represent the two colors. To give a meaning to the measured dispersion  $D_{S_0}$ , Kay and Regier created some “new” datasets  $S_i$  ( $i = 1, 2, \dots, 1,000$ ) by random rotation of the original set  $S_0$ , and measured the dispersion of each new set  $D_{S_i}$ . The “human” dispersion appears to be distinct from the histogram of the “random” dispersions with a probability larger than 99.9%. As shown in Fig. 3A of (13), the average dispersion of the random datasets,  $D_{\text{neutral}}$ , is 1.14 times larger than the dispersion of human languages.

**The Just Noticeable Difference.** Humans view the world in a nonuniform way; they have different perceptual precisions for stimuli with different wavelengths from a given continuous hue space (39, 40). Psychophysicists define the JND as a function of wavelength to describe the minimum distance at which two stimuli from the same scene can be discriminated. In principle, this parameter can either be taken as constant across the whole perceptual interval or be modulated to account for the regions with different resolution powers. Based on the curve reported in (40), we build up a human JND function as shown in Fig. 3, compared with the uniform JND.

**Details of the Simulated Model.** In the Category Game (23), at each time step two agents (a speaker and a hearer) are picked up to conduct a language game, during which the mechanisms for interaction and bargaining are as follows. A scene containing ( $M \geq 2$ ) stimuli is presented to these agents; each pair  $(x, y)$  of the stimuli must be at a distance larger than  $d_{\min}(x)$ . One of the stimuli, known to the speaker only, is the topic. The speaker checks if there is a perceptual category in which only the topic lies. If two (or more) stimuli lie in the category assigned to the topic, the speaker divides it into two (or more) new categories, each inheriting the words associated to the original category and acquiring a new word; this process is called “discrimination” (21, 23). After that, the speaker utters the most relevant name of the category containing the topic only (the most relevant name is the last name used in a winning game or the new name if this category is newly created). If the hearer does not have a category with that name, the game fails. If the hearer recognizes the name and has some stimuli in a category that is associated with it in her inventory, she picks randomly one of these stimuli (if  $M$  is not large, the hearer typically has a single candidate, see (23)). If the picked candidate is the topic, the game succeeds; otherwise, it fails. In case of failure, the hearer learns the name used by the speaker for the topic’s category; in case of success, that name becomes the most relevant name for the used category in both agents and all the other competing names in that category are removed. The full algorithmic description of the model is provided in *SI Text*.

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<sup>†</sup>This remains true when taking account of the whole dispersion histogram of the “human worlds”, as discussed in *SI Text*.

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